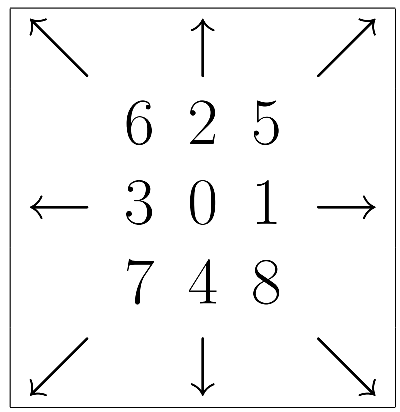
# Lattice Boltzmann

Fall Semester

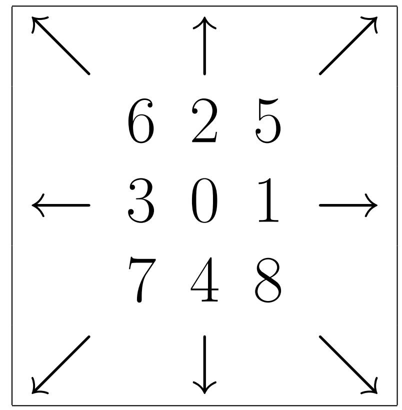
# Geometry D2Q9



# Grid

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | LY | | | | | | |
| | ⋮ | | | | | | |
| | 4 | | | | | | |
| | 3 | | | | | | |
| | 2 | | | | | | |
| | 1 | | | | | | |
| y | | 1 | 2 | 3 | 4 | . . . | LX |
| | x | | | | | | |

```
    :   |       |       |
  2dx   +-------+-------+--
        |       |       |
        |   *   |   *   |
        |       |       |
   dx   +-------+-------+--
        |       |       |
        |   *   |   *   |
        |       |       |
  0.0   +-------+-------+--
y      0.0      dx     2dx..
  x
```



# Vectors

$x_i$

| | | | | |
|---|---|---|---|---|
| ↖ | | ↑ | | ↗ |
| | -1 | 0 | 1 | |
| ← | -1 | 0 | 1 | → |
| | -1 | 0 | 1 | |
| ↙ | | ↓ | | ↘ |

$y_i$

| | | | | |
|---|---|---|---|---|
| ↖ | | ↑ | | ↗ |
| | 1 | 1 | 1 | |
| ← | 0 | 0 | 0 | → |
| | -1 | -1 | -1 | |
| ↙ | | ↓ | | ↘ |

# Weights

$$w_i \; \boxed{\begin{array}{ccccc} \nwarrow & & \uparrow & & \nearrow \\ & 1 & 4 & 1 & \\ \leftarrow & 4 & 16 & 4 & \rightarrow \\ & 1 & 4 & 1 & \\ \swarrow & & \downarrow & & \searrow \end{array}} \qquad \hat{w}_i \; \boxed{\begin{array}{ccccc} \nwarrow & & \uparrow & & \nearrow \\ & 1/36 & 1/9 & 1/36 & \\ \leftarrow & 1/9 & 4/9 & 1/9 & \rightarrow \\ & 1/36 & 1/9 & 1/36 & \\ \swarrow & & \downarrow & & \searrow \end{array}}$$

# Parameters, for example

- $\Delta x = 0.004$
- $\nu = 0.0025$
- $\omega = 1.5$
- 
- $\Delta t = \frac{(\Delta x)^2}{\nu} \cdot \frac{2-\omega}{6\omega}$
- $c = \Delta x / \Delta t$
- $\sigma = 5/12$
- $\lambda = 1/3$
- $\gamma = 1/12$

# Distribution, $g$

| | | | | |
|---|---|---|---|---|
| ↖ | | ↑ | | ↗ |
| | 6 | 2 | 5 | |
| ← | 3 | 0 | 1 | → |
| | 7 | 4 | 8 | |
| ↙ | | ↓ | | ↘ |

| | | | | |
|---|---|---|---|---|
| ↖ | | ↑ | | ↗ |
| | g[6] | g[2] | g[5] | |
| ← | g[3] | g[0] | g[1] | → |
| | g[7] | g[4] | g[8] | |
| ↙ | | ↓ | | ↘ |

- At first just read all the $g$'s from our **`gfield.txt`** file.
- In this case the grid size $LX \times LY$ is $250 \times 250$.

## Flow Field

- Velocity components sum over each $g$.

$$v_x = \sum_i \left(cx_i g_i\right)$$
$$v_y = \sum_i \left(cy_i g_i\right)$$

- Incompressible. Pressure also a sum.

$$p = \frac{c^2}{4\sigma} \cdot \left( \frac{1.5\hat{w}_0 \left(v \cdot v\right)}{c^2} + \sum_{i \neq 0} g_i \right)$$

## Cellular Automata

- Later we will calculate equilibrium $g^0$ from $v$ and $p$.

$$
\begin{aligned}
e_i &= \langle x_i, y_i \rangle \\
s_i(v) &= w_i \left( 3\frac{e_i \cdot v}{c} + 4.5\frac{(e_i \cdot v)^2}{c^2} - 1.5\frac{v \cdot v}{c^2} \right) \\
g_i^0 &= \{-4\sigma, \lambda, \lambda, \lambda, \lambda, \gamma, \gamma, \gamma, \gamma\} \frac{p}{c^2} + s_i(v)
\end{aligned}
$$

- Then (over)relax toward steady-state: $\Delta g = \omega \left(g^0 - g\right)$
- And stream (easy) plus boundary conditions (not easy).

## Visualize Streamlines

- Output $x$, $y$, $v_x$, and $v_y$ at each cell-centered point.
- Matlab script:

```
X =load( ‘xfield.txt’);
Y =load( ‘yfield.txt’);
VX=load(‘vxfield.txt’);
VY=load(‘vyfield.txt’);
%
% s = stream points
%
streamline(X,Y,VX,VY,sx,sy);
```

## Lab Assignment: Lattice Boltzmann

- No explicit Calculus in the method.
- Straightforward to parallelize.