Triangle

$$P_0 = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} P_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} P_2 = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \hat{\mathbf{n}} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$$

Equations

$$\begin{aligned} \hat{\mathbf{n}} \cdot (P - P_0) = \hat{\mathbf{n}} \cdot P - \hat{\mathbf{n}} \cdot P_0 &= 0 \\ (n_x x + n_y y + n_z z) - (n_x x_0 + n_y y_0 + n_z z_0) &= 0 \\ ax + by + cz + d &= 0 \end{aligned}$$

$$P = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} e_x + r_x t \\ e_y + r_y t \\ e_z + r_z t \end{pmatrix} E = \begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} \hat{\mathbf{r}} = \begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix}$$

Distance

$$t = \frac{\hat{\mathbf{n}} \cdot (P_0 - E)}{\hat{\mathbf{n}} \cdot \hat{\mathbf{r}}} = -\left(\frac{d + \hat{\mathbf{n}} \cdot E}{\hat{\mathbf{n}} \cdot \hat{\mathbf{r}}}\right) = -\left(\frac{d + n_x e_x + n_y e_y + n_z e_z}{n_x r_x + n_y r_y + n_z r_z}\right)$$

Interior

$$P = P_0 + a\,(P_1 - P_0) + b\,(P_2 - P_0)$$
$$a \geq 0,\ b \geq 0,\ a + b \leq 1$$

$$\mathbf{w} = P - P_0,\ \mathbf{u} = P_1 - P_0,\ \mathbf{v} = P_2 - P_0$$

$$\begin{aligned} \mathbf{w} &= a\mathbf{u} + b\mathbf{v} \\ \mathbf{w} \cdot (\mathbf{u} - (\mathbf{u} \cdot \hat{\mathbf{v}})\,\hat{\mathbf{v}}) &= (a\mathbf{u} + b\mathbf{v}) \cdot (\mathbf{u} - (\mathbf{u} \cdot \hat{\mathbf{v}})\,\hat{\mathbf{v}}) \\ \mathbf{w} \cdot (\mathbf{u} - (\mathbf{u} \cdot \hat{\mathbf{v}})\,\hat{\mathbf{v}}) &= a\mathbf{u} \cdot (\mathbf{u} - (\mathbf{u} \cdot \hat{\mathbf{v}})\,\hat{\mathbf{v}}) \\ \mathbf{w} \cdot \mathbf{u} - (\mathbf{u} \cdot \hat{\mathbf{v}})\,(\mathbf{w} \cdot \hat{\mathbf{v}}) &= a\,(\mathbf{u} \cdot \mathbf{u} - (\mathbf{u} \cdot \hat{\mathbf{v}})\,(\mathbf{u} \cdot \hat{\mathbf{v}})) \\ (\mathbf{w} \cdot \mathbf{u})\,(\mathbf{v} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{v})\,(\mathbf{w} \cdot \mathbf{v}) &= a\,((\mathbf{u} \cdot \mathbf{u})\,(\mathbf{v} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{v})\,(\mathbf{u} \cdot \mathbf{v})) \end{aligned}$$

$$a = \frac{(\mathbf{w} \cdot \mathbf{u})\,(\mathbf{v} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{v})\,(\mathbf{w} \cdot \mathbf{v})}{(\mathbf{u} \cdot \mathbf{u})\,(\mathbf{v} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{v})^2}$$

$$b = \frac{(\mathbf{w} \cdot \mathbf{v})\,(\mathbf{u} \cdot \mathbf{u}) - (\mathbf{u} \cdot \mathbf{v})\,(\mathbf{w} \cdot \mathbf{u})}{(\mathbf{u} \cdot \mathbf{u})\,(\mathbf{v} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{v})^2}$$